verse dimension of mean eddy at wall, m; $y^+ = y/l_*$, dimensionless coordinate; $u^+ = u/u_*$, dimensionless velocity; v, injection velocity, m/sec; $v^+ = v/u_*$, dimensionless injection parameter; k, roughness height, m. Subscripts: *, flow parameters for $y^+ = 1$; δ, flow parameters for $y = \delta$; W, wall parameter; s, flow parameter at rough surface with $\Psi = 1$; 0, initial flow conditions; v, flow conditions corresponding to injection velocity v.

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HYDRODYNAMIC REACTION TO ACCELERATED ONE-DIMENSIONAL MOTION OF GAS BUBBLES OF VARIABLE VOLUME IN AN UNBOUNDED LIQUID

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A solution is given for the problem of the hydrodynamic reaction of an ellipsoidal gas bubble of variable volume to accelerated motion and the relation between the value of the apparent mass and the eccentricity of the bubble.

In [1] the problem of the motion of gas bubbles with constant velocity in an unbounded volume of liquid was solved, and a relation was established between the constant velocity at which the bubble rises to the surface, the form of the bubble, and the forces of viscous friction. In a number of technical devices, motion with variable velocity and variable bubble volume is realized. The variability of the velocity leads to additional resisting forces [2]; according to the available literature, the magnitude of these forces as applied to the variable volume acting on a gas bubble in the form of an oblate ellipsoid of revolution has not been determined.

In an ellipsoidal coordinate system placed with center of mass of the bubble floating with velocity U,

$$x = c\,\mathrm{ch}\,\xi \cos\eta \cos\psi;\ y = c\,\mathrm{ch}\,\xi \cos\eta \sin\psi;\ z = c\,\mathrm{sh}\,\xi \sin\eta \tag{1}$$

for nonsymmetric potential motion the general solution for the velocity potential has the form [1]

$$\varphi(\xi,\ \eta) = [Ai\,\mathrm{sh}\,\xi - B(\mathrm{sh}\,\xi\,\mathrm{arc\,tg}\,\mathrm{sh}\,\xi + 1)]\sin\eta. \tag{2}$$

Here $i = \sqrt{-1}$, and the unknown coefficients A and B are determined from the boundary conditions

$$\lim_{\xi\to\infty} \frac{1}{c\sqrt{\mathrm{ch}^2\xi - \cos^2\eta}}\ \frac{\partial\varphi(\xi,\ \eta)}{\partial\xi} = U\sin\eta, \tag{3}$$

$$\left.\frac{\partial\varphi(\xi,\ \eta)}{\partial\vec{n}}\right|_{\xi_0} = c\,\frac{\partial\xi_0}{\partial t}\sqrt{\mathrm{ch}^2\xi_0 - \cos^2\eta}. \tag{4}$$

In Eq. (4) and below, the index 0 denotes points that belong to the surface of the bubble.

If we take into account (3) and (4), the expression for the velocity potential acquires the form

$$\varphi(\xi,\ \eta) = \frac{Uc}{L}\left[\left(L - \frac{\pi}{2}\right)\mathrm{sh}\,\xi + \mathrm{sh}\,\xi\,\mathrm{arc\,tg}\,\mathrm{sh}\,\xi + 1\right]\sin\eta + \frac{c^2}{L}\,\frac{\partial\xi_0}{\partial t}\,\frac{\mathrm{ch}^2\xi_0 - \cos^2\eta}{\mathrm{ch}\,\xi_0}\left[\frac{\pi}{2}\,\mathrm{sh}\,\xi - \mathrm{sh}\,\xi\,\mathrm{arc\,tg}\,\mathrm{sh}\,\xi - 1\right] \tag{5}$$

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For brevity in writing, in (5) we introduce the notation

$$L = \frac{\pi}{2} - \operatorname{arc\,tg\,sh} \xi_0 - \frac{\operatorname{th} \xi_0}{\operatorname{ch} \xi_0}. \tag{6}$$

The first term in (5) coincides with the expression for the velocity potential of a gas bubble of constant volume [1], and the second describes effects that arise with change in the volume.

If the velocity varies in time, then on the bubble there will act forces of inertia due to the variation of the near field of flow of the liquid. The near-field potential is given by the equation

$$\varphi_n(\xi, \eta) = \varphi(\xi, \eta) - \lim_{\xi \to \infty} \varphi(\xi, \eta) = \Phi_1(\xi, \eta) + \Phi_2(\xi, \eta), \tag{7}$$

where

$$\Phi_1(\xi, \eta) = -\frac{Uc}{L}\left(\frac{\pi}{2} \operatorname{sh} \xi - \operatorname{sh} \xi \operatorname{arc\,tg\,sh} \xi - 1\right) \sin \eta, \tag{8}$$

$$\Phi_2(\xi, \eta) = \frac{\partial \xi_0}{\partial t} \frac{c^2}{L} \frac{\operatorname{ch}^2 \xi_0 - \cos^2 \eta}{\operatorname{ch} \xi_0}\left(\frac{\pi}{2} \operatorname{sh} \xi - \operatorname{sh} \xi \operatorname{arc\,tg\,sh} \xi - 1\right). \tag{9}$$

Equation (8) describes the near field of the velocities for accelerated motion of the center of mass of the bubble and completely agrees with that obtained earlier in [1], and $\Phi_2(\xi, \eta)$ is the velocity field of the liquid that arises for motion of the bubble boundary normal to the surface.

It is obvious that the distribution of the potential $\Phi_2(\xi, \eta)$, which is symmetric with respect to the z and $r = \sqrt{x^2 + y^2}$ axes, leads to forces acting on an element of the bubble surface that are also symmetric with respect to the coordinate axes. Therefore, the resultant of these forces applied to the center of mass of the bubble will equal zero. Nevertheless the change in potential $\Phi_2(\xi, \eta)$ in time change a change in the free energy of the liquid by an amount [2]

$$T_2 = -\frac{\rho}{2} \iint_s \Phi_2(\xi_0, \eta) \frac{\partial \Phi_2(\xi_0, \eta)}{\partial \vec{n}} ds. \tag{10}$$

Here $\partial \Phi_2(\xi_0, \eta)/\partial \vec{n}$ is the normal component of the velocity of the surface;

$$\frac{\partial \Phi_2(\xi_0, \eta)}{\partial \vec{n}} = c \frac{\partial \xi_0}{\partial t} \sqrt{\operatorname{ch}^2 \xi_0 - \cos^2 \eta}, \tag{11}$$

$$ds = 2\pi c^2 \operatorname{ch} \xi_0 \cos \eta \sqrt{\operatorname{ch}^2 \xi_0 - \cos^2 \eta}. \tag{12}$$

With account of (11) and (12), Eq. (10) takes the form

$$T_2 = -2\pi\rho c^5 \left(\frac{\partial \xi_0}{\partial t}\right)^2 \operatorname{sh}^5 \xi_0 \left(1 - \frac{1}{L \operatorname{ch}^2 \xi_0 \operatorname{sh} \xi_0}\right)\left(1 - \frac{2}{3 \operatorname{sh}^2 \xi_0} + \frac{1}{5 \operatorname{sh}^4 \xi_0}\right). \tag{13}$$

Thus, the quantity (13) must be calculated in problems connected with the equation of energy balance.

The asymmetry with respect to the $r = c \operatorname{ch} \xi \cos \eta$ axis of the velocity potential $\Phi_1(\xi, \eta)$ leads to the appearance of nonzero resultant forces of reaction F_r of the liquid to the accelerated motion of the bubble. In accordance with [2], F_r is determined as the derivative with respect to time of the momentum of the liquid B_1 surrounding the bubble:

$$B_1 = -\rho \iint_s \Phi_1(\xi_0, \eta) \vec{n}\, ds, \tag{14}$$

where $\Phi_1(\xi_0, \eta)$ is the magnitude of the potential $\Phi_1(\xi, \eta)$ on the boundary of the bubble with the liquid; $\vec{n}$ is the vector of the normal to the bubble surface;

$$n = \frac{\operatorname{ch} \xi_0 \sin \eta}{\sqrt{\operatorname{ch}^2 \xi_0 - \cos^2 \eta}}. \tag{15}$$

As a result of substitution of (12) and (15) into (14) and subsequent integration, the magnitude of the momentum is determined from the expression

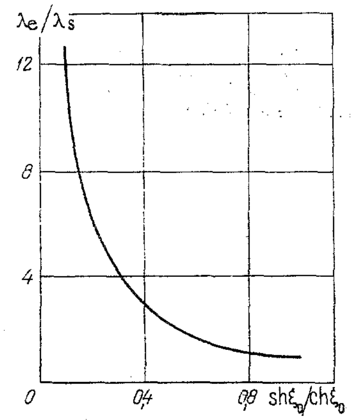

Fig. 1. Dependence of the apparent mass of an ellipsoidal bubble of constant volume on the ratio of the axes of the ellipse. The quantity λ_e/λ_s is the ratio of the apparent masses of an ellipsoid of revolution and of a sphere; $\mathrm{sh}\,\xi_0/\mathrm{ch}\,\xi_0$ is the ratio of the minor and major semiaxes of the ellipse.

$$B_1 = \frac{4}{3}\pi\rho c^3 U\left(\mathrm{sh}\,\xi_0\,\mathrm{ch}^2\,\xi_0 - \frac{1}{L}\right), \tag{16}$$

where the quantity

$$\lambda_e = -\frac{4}{3}\pi\rho c^3\left(\mathrm{sh}\,\xi_0\,\mathrm{ch}^2\,\xi_0 - \frac{1}{L}\right) \tag{17}$$

determines the apparent mass of the liquid for motion of an oblate ellipsoid of revolution.

For $c \to 0$, $\xi_0 \to \infty$, and the condition $c\,\mathrm{ch}\,\xi_0 = R$, the ellipsoid of revolution is transformed into a sphere of radius R, and (17) is transformed into

$$\lim_{\substack{c\to 0\\ \xi\to\infty}} \lambda_e = \frac{2}{3}\pi\rho R^3 = \lambda_s. \tag{18}$$

Thus, in the limit, Eq. (17) also describes the apparent mass of a sphere.

The reaction force of the liquid F_r to the accelerating bubble is determined by the equation

$$F_r = \frac{\partial}{\partial t}(\lambda_e U). \tag{19}$$

From (19) it follows that the reaction force of the liquid to its own action tends to retard the change in velocity and bubble volume. The magnitude of the force depends considerably on the form of the gas bubble and the rate of change of volume. As is shown by an analysis of Eq. (19), for fixed bubble volume, owing to only a change in the apparent mass, the reaction force is changed from a value equal to the reaction force of the liquid on a spherical bubble to an infinitely large value. The latter corresponds to the transformation of the bubble into a plane of infinite extent. The dependence of the apparent mass of the bubble on its form is illustrated in Fig. 1. The ratio of the minor axis of the ellipse to the major axis is plotted along the abscissa, and along the ordinate we plot the ratio of the apparant mass of the ellipsoid of revolution to the apparent mass of the sphere of the same volume, i.e., we have a graph of the function

$$\frac{\lambda_e}{\lambda_s} = 2\left(1 - \frac{1}{L\,\mathrm{ch}^2\,\xi_0\,\mathrm{sh}\,\xi_0}\right), \tag{20}$$

obtained by division of Eqs. (17) and (18) for the condition $V_e = V_s = \mathrm{const}$. Figure 1 shows that the generally assumed idealization of gas bubbles as spheres can cause sizable errors in a quantitative analysis of the motion of gas–liquid mixtures.

NOTATION

$\varphi(\xi, \eta)$, velocity potential; V, velocity of motion of the bubble with respect to the liquid; $\Phi(\xi, \eta)$, velocity potential of the near field; T_2, kinetic energy of the liquid due to the variation in bubble volume; F_r, reaction force of the liquid; B_1, momentum of the liquid; λ_e, apparent mass of the ellipsoid; ρ, density of the liquid; V_e, volume of the ellipsoidal bubble.

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BREAKUP OF AN ANOMOLOUSLY VISCOUS LIQUID FILM IN A CENTRIFUGAL FORCE FIELD

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An equation is obtained for the breakup radius with consideration of tipping moments and Laplacian pressure forces acting on the liquid ridge at the critical point.

As is well known, in centrifugal atomizers breakup of the liquid film and droplet formation may occur beyond the edges of the cup, at its boundary, or on its surface. Experiments have shown that in the last case the droplet dispersion becomes more homogeneous. Study of liquid film breakup is also necessary to determine the minimum liquid flow density in film-type centrifugal devices [1].

The goal of the present study is to determine the critical breakup parameters (liquid film radius of depth) as functions of the technological parameters.

We will consider the breakup of a laminar isothermal film of an anomolously viscous liquid which obeys a power-type law on the surface of a curvilinear cup. Experiments have shown that a liquid ridge is formed at the boundary between the dry and wetted surface areas. We will describe the forces acting on the liquid ridge at the critical point G using the notation of [2] (Fig. 1). We assume that the ridge has a cylindrical surface with constant radius of curvature R_r.

Considering the phenomenon of wetting angle hysteresis (i.e., the possibility of short-term rotation of the liquid film surface about the critical point G), we write the equation for the equilibrium state of the ridge

$$M_\sigma + M_v + M_\omega = 0, \tag{1}$$

where

$$M_\sigma = \sigma\delta_c; \tag{2}$$

$$M_v = -\int_0^{\delta_c} \frac{\rho v_l^2}{2}\,\delta d\delta; \tag{3}$$

$$M_\omega = -\int_0^{h_0} \rho a\omega^2 r \sin\alpha\, h dh. \tag{4}$$

Assuming that the velocity profile is defined [3, 4] as

$$v_l = \beta\left(\frac{\omega r}{4}\right)\left(\frac{2n+1}{n+1}\right)\left[1 - \left(1 - \frac{\delta}{\delta_c}\right)^{\frac{n+1}{n}}\right], \tag{5}$$

we integrate Eq. (3), obtaining

$$M_v = -T_1\delta_c^2\left(\frac{\omega r}{4}\right)^2\beta^2, \tag{6}$$

where

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